Exp:.

Find Laplace Transform of-

$$f(t) = \begin{cases} t & 1 < t < 2 \\ 4-t & 2 < t < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Sol^n:. From the definition of Laplace-

$$F(s) = L\{f(t)\} = \int_0^\infty e^{-st}\cdot f(t)\,dt$$

or $$F(s) = \int_0^1 e^{-st}\cdot f(t)\,dt + \int_1^2 e^{-st}\cdot f(t)\,dt + \int_2^3 e^{-st}\cdot f(t)\,dt + \int_3^\infty e^{-st}\cdot f(t)\,dt$$

or $$F(s) = 0 + \int_1^2 e^{-st}\cdot t\,dt + \int_2^3 e^{-st}\cdot(4-t)\,dt + 0$$

or $$F(s) = \int_1^2 e^{-st}\cdot t\,dt + \int_2^3 (4-t)\cdot e^{-st}\,dt$$

By Using Integral By-Part.

or $$F(s) = \left[\frac{t\cdot e^{-st}}{-s} - \int \frac{e^{-st}}{-s}\,dt\right]_1^2 + \left[\frac{(4-t)\,e^{-st}}{-s} - \int \frac{-1\cdot e^{-st}}{-s}\,dt\right]_2^3$$

or $$F(s) = \left[\frac{t\,e^{-st}}{-s} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)\right]_1^2 + \left[\frac{(4-t)\,e^{-st}}{-s} - \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)\right]_2^3$$

$$= \left[\frac{2e^{-2s}}{-s} - \frac{e^{-2s}}{s^2} - \frac{1\cdot e^{-s}}{-s} - \frac{e^{-s}}{s^2}\right]$$

$$F(s) = \left[\frac{t e^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_1^2 + \left[\frac{(4-t)e^{-st}}{-s} + \frac{e^{-st}}{s^2}\right]$$

$$F(s) = \left[\frac{2e^{-2s}}{-s} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2}\right] + \left[\frac{e^{-3s}}{-s} + \frac{e^{-3s}}{s^2} +\right.$$

$$F(s) = \frac{-2e^{-2s}}{s^2} + \left(\frac{1}{s} + \frac{1}{s^2}\right)e^{-s} + \left(\frac{1}{s^2} - \frac{1}{s}\right)e^{-3s}$$

$$\boxed{F(s) = \frac{(s+1)}{s^2} \cdot e^{-s} - \frac{2}{s^2}e^{-2s} + \frac{(1-s)}{s^2}e^{-3s}}$$

Exercise:

1). $f(t) = \begin{cases} \cos 3t & 0 < t < 2 \\ 0 & t > 2 \end{cases}$

2) $f(t) = \begin{cases} 4 & 0 < t < 2 \\ 2t & t > 2 \end{cases}$